Mrs Choong
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Mrs Stock
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Name:

Teacher's Name:

3U ASS II
2003 PLC

PYMBLE LADIES' COLLEGE

MATHEMATICS EXTENSION 1

YEAR 12

11th June, 2003

TIME ALLOWED – 75 Minutes

MARKING GUIDELINES:
THE MARKS FOR EACH PART ARE INDICATED BESIDE THE QUESTION

Instructions:

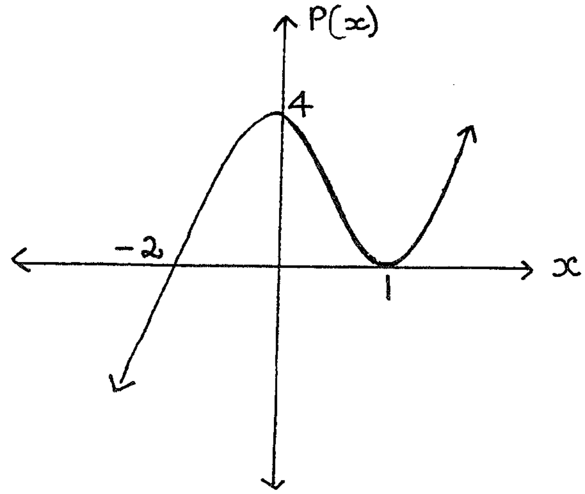
- **All questions should be attempted.**
- **All necessary working must be shown.**
- **Start each question on a new page**
- **Put you name and your teacher's name on each page**
- **Marks may be deducted for careless or untidy work.**
- **Approved calculators may be used.**
- **DO NOT staple different questions together.**
- **All rough working paper must be attached to the end of the last question.**
- **A standard integral sheet is attached.**
- **Staple a coloured sheet of paper to the back of each question.**
- **Hand in this question paper with your answers.**
- **There are four (4) questions in this paper.**

Question 1 **13 Marks**

(a) Differentiate $\tan^{-1} 2x$ 1 mark

(b) Evaluate $\int_{2\cdot5}^{5} \frac{dx}{\sqrt{25-x^2}}$ 2 marks

(c)



P(x) is sketched above

(i) Write down the equation of *P(x)* 1 mark
(Leave in factored form)

(ii) Solve $P(x) > 0$ 1 mark

(d) (i) Show that the function $f(x) = x^3 - x^2 - x - 1$ has a zero between 1 and 2. 2 marks

(ii) Taking $x = 2$ as the first approximation to this zero use Newton's method to find a second approximation. 3 marks

(e) Find the exact value of $\sin\left(2\tan^{-1}\frac{1}{\sqrt{5}}\right)$ 3 marks

Question 2 **(Start a new page)** **13 Marks**

(a) If α, β, λ are the roots of $x^3 + 6x^2 + 3x - 10 = 0$

find (i) $\alpha + \beta + \lambda$ 1 mark

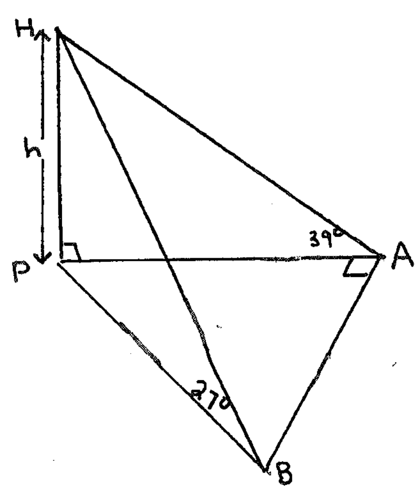
(ii) $\alpha\beta + \alpha\lambda + \beta\lambda$ 1 mark

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\lambda}$ 1 mark

(iv) $\alpha^2 + \beta^2 + \lambda^2$ 2 marks

(b) If a polynomial *P(x)* is divided by $(x-2)$ the remainder is 4.
When *P(x)* is divided by *(x + 1)* the remainder is –5.
Find the remainder when *P(x)* is divided by $(x-2)(x+1)$ 4 marks

(c)



At a point *A* due east of a vertical hill *PH*, height 'h' metres, the angle of elevation of the top of the hill is $39°$. At a point *B* 500m due south of *A* the angle of elevation of the same hill is $27°$.

(i) Show $PB = h\tan 63°$ 1 mark

(ii) Hence or otherwise find 'h' to the nearest (m). 3 marks

Question 3 **(Start a new page)** **13 Marks**

(a) Prove $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\frac{\pi}{4}$

without using calculator 2 marks

(b) The roots of $4x^3+32x^2+79x+60=0$ are α, β and $\alpha+\beta$

(i) Show $\alpha+\beta=-4$ 1 mark

(ii) Hence or otherwise solve $4x^3+32x^2+79x+60=0$ 3 marks

(c) A function is defined as $f(x)=x^3+9x^2+27x+8$ for $-2\leq x\leq 1$

(i) Show $f(x)$ is an increasing function for the given domain 2 marks

(ii) Explain why $f^{-1}(x)$ exists for $-2\leq x\leq 1$ and state its domain and range. 3 marks

(iii) Find the gradient of $y=f^{-1}(x)$ at the point $(-11,-1)$ 2 marks

Question 4 **(Start a new page)** **13 marks**

(a) (i) Sketch $y = 2\sin^{-1} 3x$ 3 marks

(ii) Shade the area bounded by $y = 2\sin^{-1} 3x$, the x-axis and $x = \frac{1}{6}$. Hence or otherwise find the exact value of $\int_0^{\frac{1}{6}} 2\sin^{-1} 3x \, dx$ 3 marks

(b) On the day his daughter was born a father invested \$1000 in a bank account that paid interest at a fixed rate of 8% per annum compounded annually.

(i) Show that there would be \$5,033.83 in the account after the payment of interest on her 21st birthday if no additional deposits were made. 1 mark

(ii) However, the father adds \$1000 to the account every year on his daughter's birthday. How much would be in the account on her 21st birthday after payment of interest and the last deposit? (the interest rate remained the same). 3 marks

(iii) Unfortunately the interest rate changed to 4% per annum compounded annually on all future investments immediately after the \$1000 deposit was made on his daughter's 10th birthday. How much would be in the account now on her 21st birthday after the payment of interest and the last deposit? 3 marks

END OF PAPER

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Solutions

Question One 13 marks

(a) $\dfrac{d\tan^{-1}2x}{dx} = \dfrac{2}{1+4x^2}$ 1

(b) $\displaystyle\int_{2.5}^{5} \frac{dx}{\sqrt{25-x^2}} = \left[\sin^{-1}\frac{x}{5}\right]_{2.5}^{5} = \sin^{-1}1 - \sin^{-1}0.5$

$= \dfrac{\pi}{3}$ 2

(c) (i) $P(x) = a(x-1)^2(x+2)$

when $x = 0$ $P(x) = 4$

$4 = 2a$

$a = 2$

$P(x) = 2(x-1)^2(x+2)$ 1

(ii) $x > 1$ $-2 < x < 1$ 1

(d) (i) $f(1) = 1-1-1-1 = -2 < 0$

$f(2) = 8-4-2-1 = 1 > 0$

As $f(x)$ is continuous and $f(1)$ and $f(2)$ have different signs $f(x)$ has at least one zero between 1 and 2 2

(ii) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$f(2) = 1$

$f'(x) = 3x^2 - 2x - 1$

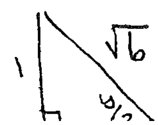
$f'(2) = 7$ 3

$= 2 - \dfrac{1}{7}$

$= \dfrac{13}{7}$

(e) let $2\tan^{-1}\dfrac{1}{\sqrt{5}} = y$ $\therefore \sin\left(2\tan^{-1}\dfrac{1}{\sqrt{5}}\right) = \sin y = 2\sin\dfrac{y}{2}\cos\dfrac{y}{2}$

$\tan^{-1}\dfrac{1}{\sqrt{5}} = \dfrac{y}{2}$



$= 2 \times \dfrac{1}{\sqrt{6}} \times \dfrac{\sqrt{5}}{\sqrt{6}}$

$\tan\dfrac{y}{2} = \dfrac{1}{\ldots}$

$= \sqrt{5}\ldots$

Question Two 13 marks

(a)

(i) $\alpha+\beta+\gamma = -6$ 1

(ii) $\alpha\beta+\alpha\gamma+\beta\alpha = 3$ 1

(iii) $\dfrac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} = \dfrac{3}{10}$ 1

(iv) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= 36 - 2\times 3$

$= 30$ 2

(b) $P(x) = Q(x)(x-2)(x+1) + ax+b$

$P(2) = 2a+b = 4$

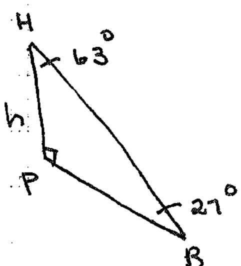
$P(-1) = -a+b = -5$

$3a = 9$

$a = 3 \quad b = -2$

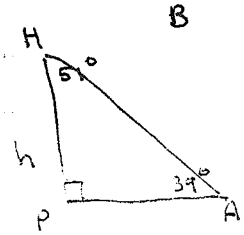
$\therefore R(x) = 3x-2$ 4

(c)

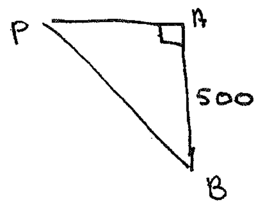


$\tan 63 = \dfrac{PB}{h}$

$AB = h\tan 63$ 1



$PA = h\tan 51°$



$PB^2 = PA^2 + AB^2$

$h^2\tan^2 63 = h^2\tan^2 51 + (500)^2$ 3

$h^2 = \dfrac{500^2}{\tan^2 63 - \tan^2 51}$

$= \dfrac{250000}{2.3265} = 107,440.513$

Question Three 13 marks

(a) let $\tan^{-1}\left(\frac{1}{4}\right) = y$ $\quad \tan^{-1}\left(\frac{3}{5}\right) = x$

$\tan y = \frac{1}{4} \quad \tan x = \frac{3}{5}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \frac{\frac{3}{5} + \frac{1}{4}}{1 - \frac{3}{5} \times \frac{1}{4}}$$

$$= 1$$

$$x + y = \tan^{-1} 1$$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{4}$$

2

(b) (i) $\alpha + \beta + \alpha + \beta = -\frac{32}{4}$

$2(\alpha+\beta) = -8$

$\alpha + \beta = -4$

1

$\therefore$ one root is -4

$$\begin{array}{r} 4x^2 + 16x + 15 \\ x+4 \;\big)\; \overline{4x^3 + 32x^2 + 79x + 60 = 0} \\ \underline{4x^3 + 16x^2} \qquad\qquad\quad \\ 16x^2 + 79x \qquad\quad \\ \underline{16x^2 + 64x} \qquad\quad \\ 15x + 60 \\ \underline{15x + 60} \end{array}$$

$\therefore (x+4)(4x^2+16x+15) = 0$

$(x+4)(2x+5)(2x+3) = 0$

$x = -4, -\frac{5}{2}, -\frac{3}{2}$

3

OR

$\alpha + \beta = -4$ $\qquad$ $\alpha\beta(\alpha+\beta) = -\frac{60}{4}$

$\alpha\beta(-4) = -\frac{60}{4}$

$\alpha\beta = \frac{15}{4}$

$\beta = \frac{15}{4\alpha}$

$\alpha + \frac{15}{4\alpha} = -4$

$4\alpha^2 + 16\alpha + 15 = 0$

$(2\alpha+5)(2\alpha+3) = 0$

$\alpha = -5/2 \ , \ -3/2$

$\beta = -3/2 \quad -5/2$

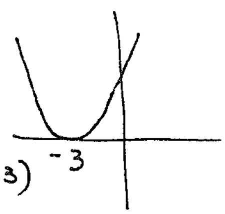
$\therefore$ Roots are $-5/2 \ , -3/2 \ . -4$

ie $\quad x = -5/2 \quad -3/2 \quad -4$ $\qquad$ (3)

(c) (i) $f'(x) = 3x^2 + 18x + 27$

$= 3(x+3)^2$

> 0 for $x > -3$
(and $x < -3$)



$\therefore f'(x) > 0$ for $-2 \leq x \leq 1$ $\qquad$ 2

(ii) As function is an increasing and continuous function for $-2 \leq x \leq 1$ it is a one-to-one function and so has an inverse. $\quad f(-2) = -18 \quad f(1) = 45$ $\qquad$ 3

D $f^{-1}(x)$: $-18 \leq x \leq 45$

R $f^{-1}(x)$ $\quad -2 \leq y \leq 1$

(iii) $f'(x) = 3(x+3)^2$

$\frac{d f^{-1}(x)}{dx} = \frac{1}{3(x+3)^2}$ $\quad$ if $x = -11$ sub in $x = -1$ and $y = -1$ $\qquad$ 2

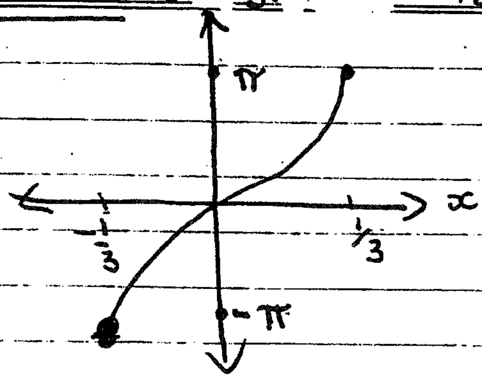
$= \frac{1}{12}$ $\qquad$ OR//

for $f^{-1}(x)$ $\qquad x = y^3 + 9y^2 + 27y + 8$ $\qquad \therefore \frac{dx}{dy} = \frac{1}{3(y+3)^2} = \frac{1}{12}$

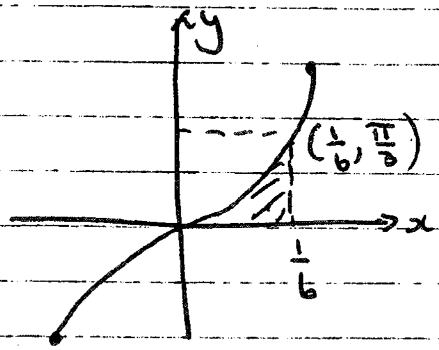
$\frac{dx}{dy} = 3(y+3)^2$

Question 4 Yr 13 marks

(a)



(3)



$y = 2\sin^{-1} 3x$

$\frac{y}{2} = \sin^{-1} 3x$

$x = \frac{1}{3}\sin\frac{y}{2}$

Area = Rectangle − area around y-axis

$= \frac{\pi}{3} \times \frac{1}{6} - \int_0^{\pi/3} \frac{1}{3}\sin\frac{y}{2}\, dy$

$= \frac{\pi}{18} + \frac{2}{3}\left[\cos\frac{y}{2}\right]_0^{\pi/3}$

3

$= \frac{\pi}{18} + \frac{2}{3}\left[\frac{\sqrt{3}}{2} - 1\right]$

b) (i) $A = 1000(1.08)^{21}$
$= \$5033.83$ 1

last deposit

(ii) $A = 1000(1.08)^{21} + 1000(1.08)^{20} \dots + \overset{\substack{\downarrow \cancel{21st} \\ \text{birthday}}}{1000}$

after on 21st birthday $= 1000\left[1 + 1.08^{1} + \dots 1.08^{21}\right]$

$= 1000\left[\dfrac{1\left[1.08^{22} - 1\right]}{1.08 - 1}\right]$ 3

$= \$55456.76$

(iii). Amount in her account after deposit on 10th birthday

$A = 1000(1.08)^{10} + 1000(1.08)^{9} + \dots 1000$

$= 1000\dfrac{1\left[1.08^{11} - 1\right]}{1.08 - 1}$

$= \$16645.49$

This now accrues interest at 4% for 11 years

$A = 16645.49 \times 1.04^{11}$

$= \underline{25624.96}$

New deposits on 11th birthday $\Rightarrow$ 21st birthday

ie 10 years

$A = 1000 \times 1.04^{10} + 1000 \times 1.04^{9} \dots 1000 \times 1.04^{1}$
$+ 1000$

$= 1000\ \dfrac{1\left[1.04^{11} - 1\right]}{1.04 - 1}$ 3

$= \underline{\$13486.35}$

Total $=$ \$39111.31